

**LO.a: Calculate and interpret major return measures and describe their appropriate uses.**

1. Siraj intends to evaluate the annualized returns of his buy-and-hold strategy after making his annual deposits to an account for each of the past three years. Which of the following methods should be used, *most* appropriately?
  - A. Geometric mean return.
  - B. Money-weighted return.
  - C. Arithmetic mean return.

2. An investor's transactions in a mutual fund and the fund's return over a three year period are given below:

	Year 1	Year 2	Year 3
New investment at the beginning of the year	\$2,000	\$2,200	\$3,000
Investment return for the year	-10%	25%	40%
Withdrawal by investor at the end of the year	\$0	\$1,000	\$0

Based on the data, the money weighted return for the investor is *closest* to:

- A. 51.8%.
  - B. 24.7%.
  - C. 74.9%.
3. An analyst observes that the historic geometric returns are 10% for fixed income securities, 5% for treasury bills and 2% for inflation. The real rate of return for fixed income securities is *closest* to:
  - A. 4.7%.
  - B. 7.2%.
  - C. 7.8%.

4. An analyst obtains the following annual rates of return for a mutual fund:

Year	2011	2012	2013
Return (%)	23	-14	-1.5

The fund's holding period return over the three-year period is *closest* to:

- A. 4.19%.
  - B. 4.75%.
  - C. 5.29%.
5. Ahmed invested in a fund which offered the following returns over the last three years:

Year	Assets under management at start of year	Net Return (%)
1.	15 million	15
2.	20 million	-5
3.	5 million	10

The money-weighted annual return is *closest* to:

- A. 4.5%.
  - B. 5.0%.
  - C. 12.1%.

6. You are evaluating the performance of two investment managers in your team:
  - Soomro: In the last 200 days, he has earned a holding period return of 9.2 percent.
  - Seemi: Over the past 5 months, her holding period return is 6.0%.
 Which manager performed better?
  - A. Soomro.
  - B. Seemi.
  - C. Both did equally well.
7. Saman purchases two shares of Sun Co, one for \$32 at time  $t = 0$  and the other for \$45 at  $t = 1$ . At  $t = 2$ , he sells them both for \$53 each. The stock paid a dividend of \$0.75 per share at  $t = 1$  and at  $t = 2$ . The periodic money weighted rate of return on the investment is *closest* to:
  - A. 23.82%.
  - B. 25.76%.
  - C. 26.75%.
8. Liquidity *least likely* impacts which of the following with respect to trading costs?
  - A. stock price.
  - B. brokerage commissions.
  - C. bid–ask spread.
9. Fred David invested in the stock of a hypothetical company called Stars Ltd. He purchased three shares worth \$100 each at the beginning of the first year. He invested in another share worth \$115 before the beginning of the second year. He sold the four shares at the end of the second year for a price of \$120 per share. At the end of each period, the stock paid a dividend of \$2 per share. Which of the following is *most likely* to be the money-weighted rate of return?
  - A. 8.76%.
  - B. 9.62%.
  - C. 10.66%.

**LO.b: Describe characteristics of the major asset classes that investors consider in forming portfolios.**

10. Which of the following asset classes have historically had the highest returns and standard deviation?
  - A. Long-term corporate bonds.
  - B. Large-cap stocks.
  - C. Small-cap stocks.
11. Which of the following asset classes have historically had the lowest returns and standard deviation?
  - A. Long term treasury bonds.
  - B. Treasury bills.
  - C. Large cap stocks.

**LO.c: Calculate and interpret the mean, variance, and covariance (or correlation) of asset returns based on historical data.**

12. The following table presents historical information for two stocks, ABC and XYZ:

Variance of returns for ABC	0.0308
Variance of returns for XYZ	0.0705
Correlation coefficient between ABC and XYZ	0.6500

The covariance between ABC and XYZ is *closest* to:

- A. 0.0014.
  - B. 0.0717.
  - C. 0.0303.
13. A measure of how the returns of two risky assets move in relation to each other is the:
- A. portfolio return.
  - B. covariance.
  - C. standard deviation.
14. Ahmed's portfolio consists of two stocks: Ivne. Ltd and Iris. Co. The standard deviation of returns is 0.25 for Ivne. Ltd and 0.14 for Iris. Co. The covariance between the returns of the two stocks is 0.0045. The correlation of returns between them is:
- A. 0.008.
  - B. 0.129.
  - C. 7.778.
15. Which of the following statements is least accurate?
- A. If you add a stock to a portfolio where the risk of the stock is equal to the risk of the portfolio and the correlation is 0.6, the overall risk of the new portfolio will be lower.
  - B. The correlation coefficient and potential benefits from diversification are inversely related.
  - C. A zero variance portfolio can be constructed by combining two securities with a correlation coefficient of 0.
16. You are a U.S investor with 78% invested in the S&P 500 and 22% invested in the Dow Jones 30 index. The risk and expected return data is given below:

	Risk (%)	Expected Return (%)	Covariance (% squared)
S&P 500	16.32%	9.82	0.43
Dow Jones 30	32.86%	14.97	

The portfolio's expected return and risk are *closest* to:

- A. 10.95% and 15.14%.
  - B. 10.90% and 15.10%.
  - C. 11.58% and 15.10%.
17. The correlation between the historical returns of Stock A and Stock B is 0.75. If the variance of Stock A is 0.25 and the variance of Stock B is 0.36, the covariance of the returns of Stock A and Stock B is *closest* to:
- A. 0.225.

- B. 0.30.
- C. 0.36.

**LO.d: Explain risk aversion and its implications for portfolio selection.**

18. An investment has a 50% probability of returning 10% and a 50% probability of returning 4%. An investor prefers this uncertain investment over a guaranteed return of 8%. This preference *most likely* indicates that the investor is risk:
- A. seeking.
  - B. neutral.
  - C. averse.
19. The risk-return relationship for a risk averse investor is *most likely* to be:
- A. positive.
  - B. negative.
  - C. neutral.
20. You are advising three clients of whom Eliyahu Goldratt is the most risk-averse. According to the utility theory, the indifference curve for Goldratt will *most likely* be the one with the:
- A. greatest slope coefficient.
  - B. smallest intercept value.
  - C. least convexity.
21. Which of the following statements about risk-averse investors is *least* accurate? A risk-averse investor:
- A. will take additional investment risk if sufficiently compensated for this risk.
  - B. seeks out the investment with minimum risk, given a certain level of return.
  - C. avoids participating in global equity markets.

**LO.e: Calculate and interpret portfolio standard deviation.**

22. Selected information about shares of two companies is provided below:

	ABC Corporation	XYZ Corporation
Standard deviation	25%	30%
Correlation of returns	0.24	
Portfolio weights	40%	60%

The standard deviation of returns of the portfolio formed with these two stocks is *closest* to:

- A. 0.0043.
  - B. 0.2259.
  - C. 0.0756.
23. An analyst studies an investment portfolio with stocks of Company ABC and Company JKL. He wishes to compute the correlation of returns between the stocks. However, the only bits of information available include the following data.

Stock	Standard Deviation	Portfolio Weights
ABC	36%	40%

JKL	27%	60%
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The standard deviation of the returns for the portfolio is 30%. The correlation coefficient for the returns is *closest* to:

- A. 0.92.
- B. 1.02.
- C. 1.84.

24. The following data is available:

Expected Return	Standard Deviation	Risk aversion coefficient
15%	27%	4

The utility of this investment is *closest* to:

- A. 0.0040.
- B. 0.0041.
- C. 0.0042.

25. If the correlation between Stock A and Stock B in a two-asset portfolio increases during a market decline, with a constant weightage of the assets and expected standard deviations of each, the portfolio's volatility will:

- A. increase.
- B. stay constant.
- C. decrease.

26. Arman is considering investing in a small-cap stock fund and a general bond fund. The correlation between the two fund returns is 0.12. Expected annual return equaled 16% and 6% respectively with standard deviation of 30% for small-cap stock and 11.5% for general bond fund. If Arman requires a portfolio return of 10 percent, the proportions in each fund respectively should be *closest* to:

- A. 30% and 70%.
- B. 36.4% and 63.4%.
- C. 40% and 60%.

27. Information about a portfolio that consist of two assets is provided below:

Asset	Portfolio Weight	Standard deviation
ABC	30%	10%
JKL	70%	8%

If the correlation coefficient between the two assets is 0.8, the standard deviation of the portfolio is *closest* to:

- A. 8.2%.
- B. 9.8%.
- C. 9.1%.

**LO.f: Describe the effect on a portfolio's risk of investing in assets that are less than perfectly correlated.**

28. Assume that two securities that are present in equal proportions in an investor's portfolio have the same expected returns and volatility. For which of the following correlations between the two securities would the investor *most likely* be able to achieve the greatest diversification benefit?
- +0.86.
  - 0.86.
  - 0.00.

29. A correlation matrix of the returns for securities A, B, and C is reported below:

Security	A	B	C
A	1.0		
B	-1.0	1.0	
C	0.5	-0.5	1.0

Assuming that the expected return and the standard deviation of each security are the same, a portfolio consisting of an equal allocation of which two securities will be *most effective* for portfolio diversification? Securities:

- A and B.
  - A and C.
  - B and C.
30. A portfolio contains equal weights of two securities that have the same standard deviation. If the correlation between the returns of the two securities was to increase, the portfolio risk would *most likely*:
- increase.
  - remain the same.
  - decrease.

**LO.g: Describe and interpret the minimum-variance and efficient frontiers of risky assets and the global minimum-variance portfolio.**

31. The set of risky portfolio that give the highest return at each level of risk will *most likely* lie on the:
- capital allocation line.
  - efficient frontier.
  - security market line.
32. The capital allocation line (CAL) dominates the efficient frontier because of the ability of the investor to:
- invest in the risk-free asset.
  - invest in market portfolio.
  - invest in a zero-beta asset.
33. Which of the following in combination with the risk-free asset forms the dominant capital allocation line?
- global minimum-variance portfolio.

- B. optimal risky portfolio.
  - C. levered portfolio of risky assets.
34. Which of the following portfolios will *most likely* lie at the point of tangency between the capital allocation line and the efficient frontier of risky assets?
- A. Optimal investor portfolio.
  - B. Global minimum variance portfolio.
  - C. Optimal risky portfolio.

**LO.h: Discuss the selection of an optimal portfolio, given an investor's utility (or risk aversion) and the capital allocation line.**

35. Relative to an investor with steep upward sloping indifference curves, an investor with a less steep indifference curve *most likely* has:
- A. a higher level of risk aversion.
  - B. a lower level of risk aversion.
  - C. the same level of risk aversion.
36. Investor X has a higher risk aversion than investor Y. On the capital allocation line, will investor Y's optimal portfolio have a higher expected return?
- A. Yes.
  - B. No, since investor Y has low risk tolerance.
  - C. No, since investor Y has high risk tolerance.
37. The optimal portfolio, as suggested by the mean–variance theory, is determined by every individual investor's:
- A. borrowing rate.
  - B. risk-free rate.
  - C. risk preference.

## Solutions

1. A is correct. The geometric mean return compounds the returns instead of the amount invested.
2. B is correct.

	Year 1	Year 2	Year 3
Starting balance (\$)	0	1,800	4,000
New investment at the beginning of the year (\$)	2,000	2,200	3,000
Net balance at the beginning of the year (\$)	2,000	4,000	7,000
Investment return for the year	-10%	25%	40%
Investment gain/loss (\$)	-200	1,000	2,800
Withdrawal by investor at the end of the year (\$)	0	1,000	0
Balance at the end of the year (\$)	1,800	4,000	9,800

CF<sub>0</sub> = -2,000

CF<sub>1</sub> = -2,200 (new investment at the beginning of year 2)

CF<sub>2</sub> = -2,000 (withdrawal of 1,000 at the end of year 2, -3,000 new investment at the beginning of year 3)

CF<sub>3</sub> = 9,800 (balance at the end of year 3)

The money weighted return can be calculated as:

CF<sub>0</sub> = -2,000, CF<sub>1</sub> = -2,200, CF<sub>2</sub> = -2,000, CF<sub>3</sub> = 9,800, CPT IRR.

IRR = 24.74%.

3. C is correct. Real rate of return =  $\frac{1 + 0.1}{1 + 0.02} - 1 = 7.8\%$
4. A is correct.  $[(1 + 0.23)(1 - 0.14)(1 - 0.015)] - 1 = 0.0419 = 4.19\%$ .
5. A is correct. All amounts are in million dollars. The table below shows the computation for cashflows at the start of every year:

Year	1	2	3
Balance from previous year	0	17.25	19
New investment at the start of the year (inflow)	15	2.75	0
Withdrawal at the start of the year (outflow)	0	0	14
Net balance at the beginning of year (given in the question)	15	20	5
Investment return for the year (given in the question)	15%	-5%	10%
Investment gain (loss)	2.25	-1	0.5
Balance at the end of year	17.25	19	5.5

In order to calculate the money-weighted return (IRR) we assume the final amount (5.5) is withdrawn. The money-weighted return is the IRR, which can be calculated using a financial calculator as follows:



CF0 = -15, CF1 = -2.75, CF2 = 14, CF3 = 5.5, CPT IRR.  
IRR = 4.52%.

6. A is correct.

Annualized return for Soomro =  $[(1 + 0.092)^{(365/200)}] - 1 = 0.174 = 17.4\%$   
Annualized return for Seemi =  $[(1 + 0.06)^{365/150}] - 1 = 0.152 = 15.2\%$

7. C is correct. The money-weighted return (IRR) can be computed using a financial calculator:

CF0 = -32; CF1 = -44.25; CF2 = 107.5; CPT IRR. IRR = 26.75%.

8. B is correct. Brokerage commissions are negotiated with the brokerage firm. A security's liquidity impacts the operational efficiency of trading costs.

9. C is correct.

**Step 1:**

Calculate the cash inflows and outflows at  $t = 0, 1$  and  $2$ .

At  $t = 0$ , Fred purchased 3 shares worth \$100 each resulting in an outflow of \$300

At  $t = 1$ , Fred purchased 1 share worth \$ 115 and thus an outflow of \$115.

Fred also received dividends worth \$6 on the shares purchased earlier and thus an inflow of \$6.

Net cash outflow of \$109

At  $t = 3$ , Fred sold 4 shares for worth \$120 each therefore an inflow of \$480.

Fred also received dividends worth \$8 resulting in a net cash inflow of \$488.

**Step 2:**

Given the following Net Cash flows, calculate the IRR which is equivalent to the money-weighted return.

CF0 = -300, CF1 = -109 and CF2 = 488.

IRR or money weighted rate of return = 10.66%.

10. C is correct. Small-cap stocks have had the highest annual return and standard deviation of return over time. Large-cap stocks and bonds have historically had lower risk and return than small-cap stocks.

11. B is correct. Treasury bills have had the lowest annual return and standard deviation of return over time.

12. C is correct.  $Cov_{ij} = \sigma_i \sigma_j r_{ij} = \sqrt{0.0308 * 0.0705} * 0.6500 = 0.0303$

13. B is correct. The covariance is defined as the co-movement of the returns of two assets or how well the returns of two risky assets move together. Range and standard deviation are measures of dispersion and measure risk, not how assets move together.

14. B is correct.  $\frac{0.0045}{0.25 * 0.14} = 0.129$ .

15. C is correct. A zero-variance portfolio can only be constructed if the correlation coefficient between assets is -1.

16. A is correct.

$$\text{Portfolio return} = (0.78 * 0.0982) + (0.22 * 0.1497) = 0.1095 = 10.95\%.$$

$$\begin{aligned} \text{Portfolio risk} &= [w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\text{Cov}(R_1, R_2)]^{1/2} \\ &= [(0.78^2 * 0.1632^2) + (0.22^2 * 0.3286^2) + (2 * 0.78 * 0.22 * 0.0043)]^{1/2} = (0.02291)^{1/2} \\ &= 15.14\%. \end{aligned}$$

17. A is correct.  $\text{Cov}_{ij} = \sigma_i \sigma_j r_{ij} = \sqrt{0.25 * 0.36} * 0.75 = 0.225$

18. A is correct. A risk seeking investor prefers more risk to less risk. In the above example, he prefers the investment with an expected return of  $(0.5)10 + (0.5)4 = 7\%$  over a guaranteed return of 8%.

19. A is correct. Historical data over long periods of time indicate that there exists a positive risk–return relationship, which is a reflection of an investor’s risk aversion.

20. A is correct. The most risk-averse investor has the indifference curve with the greatest slope.

21. C is correct. Risk-averse investors are generally willing to invest in risky investments, if the return on the investment is sufficient to reward the investor for taking on this risk. Participants in securities markets are generally assumed to be risk-averse investors.

22. B is correct. Portfolio standard deviation =

$$\sqrt{(0.25^2 * 0.4^2 + 0.3^2 * 0.6^2 + 2 * 0.24 * 0.4 * 0.6 * 0.25 * 0.3)} = 0.2259.$$

23. A is correct. The standard deviation of the returns for a portfolio is given by:

$$\begin{aligned} \sigma_{portfolio} &= \sqrt{(\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 \sigma_1 w_1 \sigma_2 w_2 \rho_{12})} \\ 0.3 &= \sqrt{(0.36^2 * 0.4^2 + 0.27^2 * 0.6^2 + 2 * 0.36 * 0.4 * 0.27 * 0.6 * \rho)} \\ \text{Solve the equation to deduce the unknown i.e. } \rho &= 0.92. \end{aligned}$$

24. C is correct.

$$U = E(r) - 0.5A\sigma^2$$

$$U = 0.15 - 0.5 * 4 * 0.27^2 = 0.0042.$$

25. A is correct. Higher correlations will result in a lower diversification benefit and higher volatility.

26. C is correct.

$$10\% = w_1 * 16\% + (1 - w_1) * 6\%;$$

$$w_1 = 40\%, (1 - w_1) = 60\%.$$

Thus, 40 percent should be invested in the small-cap fund and 60 percent should be invested in the bond fund.

27. A is correct. Portfolio standard deviation =

$$\sqrt{(0.3)^2(0.1)^2 + (0.7)^2(0.08)^2 + 2(0.3)(0.7)(0.1)(0.08)} = 0.082 = 8.2\%.$$

28. B is correct. Diversification benefit is greatest when a portfolio consists of securities that do not move together and thus the investor should invest in securities with the lowest correlation i.e.  $-0.86$ .

29. A is correct. The negative correlation of  $-1.0$  between investment instruments A and B is lowest and therefore is most effective for portfolio diversification.

30. A is correct. The standard deviation of the portfolio is directly proportional to the correlation of assets within the portfolio.

31. B is correct. The efficient frontier is the part of the minimum variance frontier which represents the set of portfolios that will give the highest return at each risk level.

32. A is correct. With the efficient frontier we are only allowed to invest in risky assets. With the CAL this constraint is relaxed and we are also allowed to invest in the risk-free asset.

33. B is correct. The use of leverage and the combination of a risk-free asset and the optimal risky asset will dominate the efficient frontier of risky assets (the Markowitz efficient frontier).

34. C is correct. The optimal risky portfolio lies at the point of tangency between the capital allocation line and the efficient frontier of risky assets.

35. B is correct. An investor with less steep indifference curves has a lower level of risk aversion.

36. A is correct. Investor Y has a low risk aversion coefficient, therefore a high risk tolerance and a higher expected return on the capital allocation line.

37. C is correct. Each individual investor's optimal mix of the risk-free asset and the optimal risky asset is determined by the investor's risk preference.